

Status of α_s determinations from the non-perturbatively renormalised three-gluon vertex

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We demonstrate the feasibility of computing α_s from the lattice three-gluon vertex in the Landau gauge. Data from 16^4 and 24^4 quenched lattices at $\beta = 6.0$ are presented. Our main result is that 2-loop asymptotic scaling is observed for momenta in the range $1.8 - 2.3$ GeV, where lattice artifacts appear to be under control.

1. THE METHOD

α_s can be extracted from the 3-gluon vertex [1] by evaluating two-point and three-point off-shell gluon Green's functions on the lattice and imposing non-perturbative renormalisation conditions on them, for different values of the external momenta. The main advantages of this method are the possibility to obtain $\alpha_s(\mu)$ at several momentum scales μ from a single simulation and the fact that lattice perturbation theory (LPTH) is not needed to match our coupling to $\alpha_{\overline{MS}}$. Also, the method can be applied with no modifications to the unquenched case. On a more technical level, additional advantages are that, since one works in momentum space, lattice artifacts can be carefully analysed, as shown in section 2.1, and since only gluonic operators are used, the technology is simple. The method is fully described in [2]. Working in the Landau gauge and using a suitable definition of the lattice gluon field A_μ , one can define the momentum space gluon propagator $G_{U\mu\nu}^{(2)}(p) \equiv T_{\mu\nu}(p) G_U(p^2)$, where $T_{\mu\nu}(p)$ is the transverse projector, and the complete gluon three-point function $G_{U\alpha\beta\gamma}^{(3)}(p_1, p_2, p_3)$. The propagator is non-perturbatively renormalised by imposing that for $p^2 = \mu^2$ it attains its continuum

tree-level value:

$$G_R(p)|_{p^2=\mu^2} = Z_A^{-1}(\mu a) G_U(pa)|_{p^2=\mu^2} = \frac{1}{\mu^2}. \quad (1)$$

The three-gluon vertex is evaluated at the asymmetric kinematical points where, based on the general form of continuum the vertex function in the Landau gauge [3], the following relation holds:

$$\frac{\sum_{\alpha=1}^4 G_{U\alpha\beta\alpha}^{(3)}(pa, 0, -pa)}{(G_U(pa))^2 G_U(0)} = 6 i Z_V^{-1}(pa) g_0 p_\beta. \quad (2)$$

By computing the above ratio one can determine the factor $Z_V^{-1} g_0$, where Z_V is the vertex renormalisation constant. Finally, one can define the running coupling g at the scale μ from the renormalised three-gluon vertex as

$$g(\mu) = Z_A^{3/2}(\mu a) Z_V^{-1}(\mu a) g_0. \quad (3)$$

and $\alpha_s(\mu) \equiv g(\mu)^2/4\pi$. This choice corresponds to a momentum subtraction scheme, usually referred to as \overline{MOM} in continuum QCD [4].

2. NUMERICAL RESULTS

We consider two quenched data sets at $\beta = 6.0$; 150 configurations on a 16^4 lattice, and 103 configurations on a 24^4 lattice. We used a Landau

gauge fixing overrelaxation algorithm, with the final iterations being performed in double precision. It is worth emphasising that the numerical accuracy of the gauge-fixing is crucial to obtain a good signal for the three-point function.

2.1. Analysis of Lattice Artifacts

A comparison of numerical values of ratios of Green's functions tensor components with the theoretical expectations from LPTH and continuum perturbation theory (CPTH) provides a check for discretisation errors. In the case of the gluon propagator, the numerical values are in perfect agreement with LPTH. As for the three-gluon vertex, in Table 1 we show various ratios of tensor components on the 16^4 lattice, where in the first column the lattice momentum components are specified. Unless otherwise noted, the uncertainty is less than one unit in the last quoted figure. Only the ratio G_{101}/G_{111} is in poor agreement with LPTH. This is related to the fact that the Landau gauge condition does not fix this ratio. Overall, the tensor structure is the one expected from LPTH. This enables us to estimate the size of the lattice artifacts and to identify a "continuum window" in momentum space.

2.2. Renormalisation Constants and Running Coupling

In Figure 1 we plot $g(\mu)$ on the 16^4 lattice. All data are plotted vs. $\mu = \sqrt{p^2}$, expressed in GeV. In order to detect violations of rotational invariance, we have used whenever possible different combinations of lattice momentum vectors for a fixed value of p^2 , plotting separately the corresponding data points.

We obtain a clear signal, although the data show some violation of rotational invariance.

The important question is whether there is a momentum range where our coupling runs according to the two-loop expression

$$\frac{1}{g^2(\mu)} = b_0 \ln\left(\frac{\mu^2}{\Lambda_{\widetilde{MOM}}^2}\right) + \frac{b_1}{b_0} \ln \ln\left(\frac{\mu^2}{\Lambda_{\widetilde{MOM}}^2}\right), \quad (4)$$

where $b_0 = 11/16\pi^2$, $b_1 = 102/(16\pi^2)^2$ and $\Lambda_{\widetilde{MOM}}$ is the QCD scale parameter for our scheme. To investigate this point, we compute $\Lambda_{\widetilde{MOM}}$ as a function of the measured values of

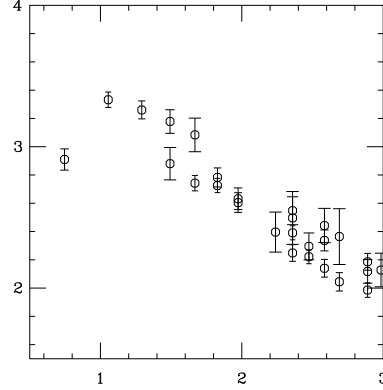


Figure 1. Running coupling $g(\mu)$ vs. μ on the smaller lattice.

$g^2(\mu)$ according to the formula

$$\Lambda_{\widetilde{MOM}} = \mu \exp\left(-\frac{1}{2b_0 g^2(\mu)}\right) [b_0 g^2(\mu)]^{-\frac{b_1}{2b_0^2}}. \quad (5)$$

If the coupling runs according to (4), then $\Lambda_{\widetilde{MOM}}$ as defined above approaches a constant value when $\mu \rightarrow \infty$. By plotting $\Lambda_{\widetilde{MOM}}$ versus μ (see Figure 2), one notices that

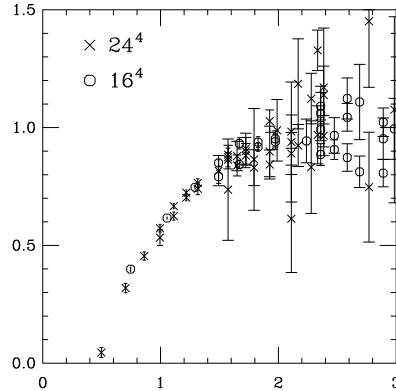


Figure 2. $\Lambda_{\widetilde{MOM}}$ vs. μ for our two lattice sizes.

for $\mu < 1.8$ GeV, $\Lambda_{\widetilde{MOM}}$ depends strongly on μ . However, in the range $1.8 < \mu < 2.3$ GeV

Table 1

Symmetry tests for $G_{U\alpha\beta\gamma}^{(3)}(p, 0, -p)$ on the 16^4 lattice at $\beta = 6.0$ (C=CPTH, L=LPTH, N=numerical).

	G_{010}/G_{111}			G_{101}/G_{111}			G_{011}/G_{111}		
	C	L	N	C	L	N	C	L	N
(1, 1, 0, 0)	1	1	1.000	1	1	1.1(2)	1	1	1.000
(1, 2, 0, 0)	4	3.848	3.848	1/2	0.510	0.3(1)	2	1.962	1.962
(2, 2, 0, 0)	1	1	1.000	1	1	0.8(3)	1	1	1.000

the data are consistent with a constant value for $\Lambda_{\widetilde{MOM}}$. No violations of rotational invariance are observed and a comparison of the two lattice sizes shows no volume dependence. For $\mu > 2.3$ GeV, rotational invariance is broken by higher order terms in a^2 and the two-loop behaviour disappears. In summary, we appear to have a “continuum window” in the range $1.8 < \mu < 2.3$ GeV, where two-loop scaling is observed and lattice artifacts are under control. In order to extract a prediction for $\Lambda_{\widetilde{MOM}}$, we fit the data points in the continuum window to the curve obtained by inserting (4) in (5). We take as our best estimate the fit to the 16^4 data, for which the statistical errors are smaller, and obtain

$$\Lambda_{\widetilde{MOM}} = 0.88 \pm 0.02 \pm 0.09 \text{ GeV}, \quad (6)$$

where the first error is statistical and the second error comes from the uncertainty on the value of a^{-1} .

3. MATCHING TO \overline{MS}

We can extract a prediction for $\alpha_{\overline{MS}}$ with zero quark flavours from our numerical results for $\Lambda_{\widetilde{MOM}}$. The ratio $\Lambda_{\overline{MS}}/\Lambda_{\widetilde{MOM}}$ can be determined to all orders in the coupling constant from a one-loop continuum calculation. We obtain, in the Landau gauge and for zero quark flavours $\Lambda_{\overline{MS}}^{(0)}/\Lambda_{\widetilde{MOM}}^{(0)} = 0.35$. This result is in agreement with previous one-loop calculations of the three-gluon vertex [5]. Using (6) and the above result, we obtain

$$\alpha_{\overline{MS}}^{(0)} = 0.31 \pm 0.05 \text{ GeV}. \quad (7)$$

This is the main result of our computation, which is in very good agreement with the one of ref. [6].

In terms of $\alpha_{\overline{MS}}^{(0)}$, our result yields:

$$\alpha_{\overline{MS}}^{(0)}(2.0 \text{ GeV}) = 0.24 \pm 0.02. \quad (8)$$

Although LPTH is not needed for matching, we can use it to perform some interesting cross-checks of the continuum calculation (see [2] for details).

4. CONCLUSIONS

We have shown in the quenched approximation that a non-perturbative determination of the QCD running coupling can be obtained from first principles by a lattice study of the three-gluon vertex. We have some evidence that systematic lattice effects are under control in our calculation. LPTH is not needed to match our results to \overline{MS} and the extension to the full theory does not present in principle any extra problem.

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REFERENCES

1. C. Parrinello, Phys. Rev. D50 (1994) 4247.
2. B. Allés et al., Nucl. Phys. B502 (1997) 325.
3. J.S. Ball, T.W. Chiu, Phys. Rev. D22 (1980) 2550.
4. A. Hasenfratz, P. Hasenfratz, Phys. Lett. B93 (1980) 165.
5. F.T. Brandt and J. Frenkel, Phys. Rev. D33 (1986) 464.
6. G.S. Bali and K. Schilling, Phys. Rev. D47 (1993) 661.